Revisions

Complex Network Theory

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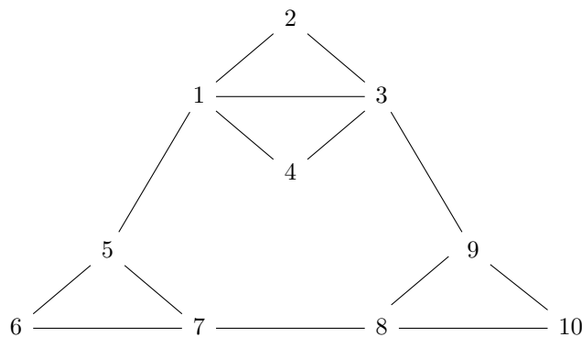

Figure 1: A Graph $G = (V, E)$.

1 Partitioning a Graph via Spectral Theory

1. Write the Laplacian matrix of the graph $G = (V, E)$ from Figure 1.
2. One of the following vectors is the Fiedler vector of G. Which one ? Justify your answer.

$$(a) : v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad (b) : v_2 = \begin{bmatrix} -1 \\ -\sqrt{2} \\ -1 \\ -\sqrt{2} \\ \sqrt{2}-1 \\ 1 \\ 1 \\ 1 \\ \sqrt{2}-1 \\ 1 \end{bmatrix}, \quad (c) : v_3 = \begin{bmatrix} 1 \\ 6/5 \\ 1 \\ 6/5 \\ 4/15 \\ 0 \\ 0 \\ -1 \\ -4/15 \\ -1 \end{bmatrix}, \quad (d) : v_4 = \begin{bmatrix} 6 \\ -4 \\ 6 \\ -4 \\ -4 \\ 1 \\ 1 \\ 1 \\ -4 \\ 1 \end{bmatrix}$$

3. Recall the formula of the isoperimetric ratio.
4. Apply the Sweep Cut method to G to partition its set of nodes (detail the steps and the computations).
5. Does the resulting partitioning verify the Cheeger's inequalities ? Justify your answer.

2 Comparing Partitionings

We denote by

$$\begin{aligned} \mathcal{P} =& \Big\{ \{1,2,3,4\}, \{5,6,7\}, \{8,9,10\} \Big\}, \\ \mathcal{C} =& \Big\{ \{1,2,3,4\}, \{5,6,7,8,9,10\} \Big\}, \\ \mathcal{K} =& \Big\{ \{1,2\}, \{3,4\}, \{5,6\}, \{7,8\}, \{9,10\} \Big\}, \end{aligned}$$

three partitionings obtained on the graph $G = (V, E)$ from Figure 1.

1. Write the agreement/disagreement tables and the confusion matrices of $\mathcal{P}$ and $\mathcal{C}$ in one hand, and $\mathcal{P}$ and $\mathcal{K}$ on the other hand.
2. Compute the ARI and the MI between $\mathcal{P}$ and $\mathcal{C}$ on one hand, and between $\mathcal{P}$ and $\mathcal{K}$ on the other hand. Discuss the results.
3. Compute the modularity and the normalised cuts of $\mathcal{P}$, $\mathcal{C}$, $\mathcal{K}$. Discuss the results.